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On the new unit Lindley distribution

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ABSTRACT
In this article it is shown that the new unit Lindley distribution of Mazuchelli et al. [2] is in fact complementary of the original unit Lindley distribution of Mazuchelli et al. [1] and as such almost all properties of the former directly follows from those of the later quite easily. Some illustrative derivation of properties of new unit Lindley distribution from those of the unit Lindley distribution is also presented.

KEYWORDS
Complementary random variable; Unit-Lindley distribution ; Exponential family

1. Introduction

Recently, [2] introduced a new unit Lindley (NUL) distribution and studied some properties and applications of the same. The authors stated it as a new distribution thus different from the original unit Lindley (UL) distribution of [1].

A random variable $X \sim$ UL with parameter θ that is UL(θ) if its probability density function (pdf) and cumulative distribution function (cdf) are respectively given by

$$f_X(x \mid \theta) = \frac{\theta^2}{1+\theta} (1-x)^{-3} \exp\left(-\frac{\theta\, x}{1-x}\right), \qquad 0 < x < 1,\, \theta > 0. \tag{1}$$

and

$$F_X(x \mid \theta) = 1 - \left(1 - \frac{\theta\, x}{(1+\theta)\,(x-1)}\right) \exp\left(-\frac{\theta\, x}{1-x}\right), \qquad 0 < x < 1,\, \theta > 0. \tag{2}$$

On the other hand, if a random variable $Y \sim$ NUL (Mazuchelli et al., 2020) with parameter θ that is NUL(θ) then its pdf and cdf are respectively given as

$$f_Y(y \mid \theta) = \frac{\theta^2}{y^3(1+\theta)} \exp\left(-\frac{\theta\,(1-y)}{y}\right), \qquad 0 < y < 1,\, \theta > 0. \tag{3}$$

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and

$$F_Y(y \mid \theta) = \frac{\theta + y}{y(1+\theta)} \exp\left(-\frac{\theta\,(1-y)}{y}\right), \qquad 0 < y < 1,\, \theta > 0. \tag{4}$$

In the next section we provide main results which show that the NUL distribution is in fact complimentary of UL distribution.

2. Main Results

Theorem 2.1. *If $X \sim UL(\theta)$, then $Y = 1 - X \sim NUL(\theta)$.*

Proof:

$$\begin{aligned}
F_Y(y \mid \theta) &= F_{1-X}(y \mid \theta) \\
&= P(1 - X \leq y) \\
&= P(X > 1 - y) \\
&= 1 - P(X \leq 1 - y) \\
&= 1 - \left[1 - \left(1 - \frac{\theta\,(1-y)}{(1+\theta)\,(-y)}\right) \exp\left(-\frac{\theta\,(1-y)}{y}\right)\right] \\
&= \frac{\theta + y}{y(1+\theta)} \exp\left(-\frac{\theta\,(1-y)}{y}\right).
\end{aligned}$$

This is the cdf of NUL(θ) in equation (4). Hence proved.

As a result of this simple relation between the UL(θ) and NUL(θ) all properties of the later can be derived directly from the corresponding properties of the former. In what follows we show derivation of some of the important properties of NUL(θ) from those of UL(θ).

Theorem 2.2. *The NUL(θ) is unimodal with mode at $Y = \frac{\theta}{3}$ for $\theta < 3$ and at $Y = 1$ for $\theta \geq 3$*

Proof: It is known from [1] that $X \sim$ UL(θ) is unimodal with mode at $X = 1 - \frac{\theta}{3}$ for $\theta < 3$ and at $X = 0$ for $\theta \geq 3$, Required result follows immediately from the fact that $Y = 1 - X$ and $0 < X \leq 1$.

Theorem 2.3. *NUL(θ) belongs to exponential family.*

Proof: It is known from [1] that the pdf of $X \sim$ UL(θ) in equation (1) belongs to exponential family

$$f_X(x \mid \theta) = \exp\left[Q(\theta)\, T_X(x) + D(\theta) + S_X(x)\right].$$

with $Q(\theta) = -\theta, T_X(x) = x/(1-x), D(\theta) = \log\left[\frac{\theta^2}{1+\theta}\right], S_X(x) = \log(1-x)^{-3}$.

Now using the transformation $Y = 1 - X$ and $0 < X \leq 1$, it is easy to verify that NUL(θ) with pdf in equation (3) can be rewritten as

$$f_Y(y \mid \theta) = \exp\left[Q(\theta)\,T_Y(y) + D(\theta) + S_Y(y)\right],$$

where $Q(\theta) = -\theta, T_Y(y) = T_X(1-y) = \frac{1-y}{y}, D(\theta) = \log\left[\frac{\theta^2}{1+\theta}\right], S_Y(y) = S_X(1-y) = \{\log(y)\}^{-3}$. Hence, belongs to the exponential family.

Theorem 2.4. *The CDF of the NUL(θ) is convex for $\theta > 3$.*

Proof: We know that UL(θ) is concave for $\theta > 3$ ([1]). Now

$$\begin{aligned}
\frac{\mathrm{d}^2}{\mathrm{dy}^2} F_Y(y \mid \theta) &= \frac{\mathrm{d}}{\mathrm{dy}} f_Y(y \mid \theta) \\
&= \frac{\mathrm{d}}{\mathrm{dy}} f_X(1-y \mid \theta) \\
&= -f_X^{/}(1-y \mid \theta) \\
&= -\frac{\mathrm{d}^2}{\mathrm{dy}^2} F_X(1-y \mid \theta)
\end{aligned}$$

But because of the concavity of UL(θ) for $\theta > 3$ we have

$$\frac{d^2}{\mathrm{dx}^2} F_X(x \mid \theta) < 0.$$

Thus for $\theta > 3$ $\frac{\mathrm{d}^2}{\mathrm{dy}^2} F_Y(y \mid \theta) > 0$. Hence proved.

Theorem 2.5. *The mean of the NUL(θ) is $\frac{\theta}{\theta+1}$.*

Proof: We know that for UL(θ) the mean is $\frac{1}{\theta+1}$ ([1]). Hence mean of Y ~NUL(θ) can be obtained as

$$E(Y) = E(1-X) = 1 - \frac{1}{\theta+1} = \frac{\theta}{\theta+1}$$

.

In fact the k^{th} moment for Y ~NUL(θ) can be written by using $E(Y^k) = E(1-X)^k$.

3. Conclusion

In this short article it is shown that the recently reported new unit Lindley distribution ([2]) is related to the original Unit Lindley distribution ([1]). All properties derived for either one can be easily translated for that of the other. Moreover the regression modeling of the mean of $Y = 1 - X$ for $X \sim$ UL(θ) distribution is equivalent to the regression modelling mean of $Y \sim$NUL(θ) distribution. As such the new unit Lindley distribution may at best be seen as a dual or complementary unit Lindley distribution. It should be noted here that for any continuous random variable X with distribution in $(0, 1)$, $Y = 1 - X$ too will have a distribution in same range.

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4. Reference

References

[1] Mazucheli, J., Menezes, A.F.B., and Chakraborty, S.(2019). On the one parameter unit-Lindley distribution and its associated regression model for proportion data. Journal of Applied Statistics, 46(4), 700-714

[2] Mazucheli, J., Bapat, S.R., Menezes, A.F.B.(2020). A new one-parameter unit-Lindley distribution. Chilean Journal of Statistics, 11(1), 53-67.